An Algorithmic Comparison between Chess and Mathematics

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Analyze the following chess position (Figure 1) (Rook-King-Queen vs. King checkmate)

Chess Algorithm (Assume that you are playing White and the black King is on the nth rank)

Algorithm 1: Black King is on first through fourth rank

Goal: Checkmate King on first rank

If the White King is in the rectangle formed between the position of the Black King and the piece(other than the White King) furthest from the King, move the King completely outside of this rectangle. After the King has moved outside the rectangle, continue the given algorithm and the new (or same) position of the King should be the nth rank.

Move 1a: Move Rook or Queen (whichever is furthest from the King) to the (n+1)st rank on a square non-adjacent to the King.

If the Rook or Queen is already on the (n+1)st rank,

Move 1b: Move *the other piece* (the piece not on the (n+1)st rank) to "check" the King provided that *the other piece* would not be on a square adjacent to the King.

If the "check" results in *the other piece* being adjacent to the King or if there is another piece on the file so that the "check" cannot happen,

Move 1c: Move *the other piece* to the file furthest from the King so that it is the only piece on that file.)

Move 2: Move *the other piece* to the nth rank.

Move 3: Move piece on (n+1)st rank to (n-1)st rank provided that the piece will not be adjacent to the King. If the piece would end up being adjacent to the King, move the piece to the file furthest from the King so that it is the only piece on that file.

Move 4: Move piece on nth rank to (n-2)nd rank provided that the piece will not be adjacent to the King. If the piece would end up being adjacent to the King, move the piece to the file furthest from the King so that it is the only piece on that file.

Move 5: Move piece on (n-1)st rank to (n-3)rd rank provided that the piece will not be adjacent to the King. If the piece would end up being adjacent to the King, move the piece to the file furthest from the King so that it is the only piece on that file.

Move 6: Move piece on (n-2)nd rank to (n-4)th rank provided that the piece will not be adjacent to the King. If the piece would end up being adjacent to the King, move the piece to the file furthest from the King so that it is the only piece on that file.

Move 7: Move piece on (n-3)rd rank to (n-5)th rank provided that the piece will not be adjacent to the King. If the piece would end up being adjacent to the King, move the piece to the file furthest from the King so that it is the only piece on that file.

A solution (checkmate) should be found after the algorithm is complete.

Mathematical Algorithm

Consider the following equation:

$$\frac{2(x-7)}{6} + \frac{4x+7-3x}{2(x+1)-3x+x} = \frac{7(x-4)}{12}$$

Use the following algorithm to solve linear equations:

Step 1: Simplify denominators

Step 2: Simplify numerators (maintain parenthesis)

Step 3: Multiply all terms by the LCD

Step 4: Use distributive property.

Step 5: Combine like terms.

Step 6: Use addition or subtraction to move variables to one side of the equation and numbers to the other side of the equation.

Step 7: Use multiplication or division to solve for the variable.

Step 8: Check answer by substituting into the original equation.

The similarities between the two algorithms are as follows.

I. Depending on the complexity or simplicity of the equation or the chess position, all steps of the algorithm are not required, but the steps should be done in order.

a) For example, solving the equation $2x + 6 = 12$ only requires steps 6, 7, and 8, but in that order. Step 7 cannot be done before step 6.

b) In the chess position below, only one move would be required to win the game. We would not need to do the other steps in the algorithm.

II. The steps in the algorithm must be clearly understood.

a) Common errors that may be made in trying to find the solution of the equation above would be (1) Incorrect use of the distributive property (2) not combining like terms or combining terms that that are not like terms (3) Not being able to find LCD (4) Not multiplying all terms by the LCD correctly (5) Incorrect use of the addition property (adding on one side but subtracting on the other side) (6) Careless mistakes in adding, subtracting, multiplying, or dividing signed integers (7) Not taking the time to check the equation.

b) In the chess position above, since black has only its King, white must win the game in 50 moves or the game is considered to be a draw (tie), common errors which would make it hard for white to win would be (1) placing a piece adjacent to the King that is not protected (if one of the pieces are captured, a solution (checkmate) can still be found but it is much harder, similar to using a common denominator instead of the least common denominator, but if both the Rook and Queen are captured, a checkmate cannot occur) (2) Not moving a piece that is being attacked by the black King (3) Moving a piece away from the King, but placing the piece on the same file as another piece (this error may increase the number of steps in the algorithm)

III. In both the equation and the chess position, an error made in the algorithm does not necessarily mean that a solution(checkmate) cannot be found.

a) In the chess position, poor moves may be made by White (allowing the King to stay in the middle of the chessboard), but a solution can be found after a while if the algorithm is followed. Also, the opponent can make poor moves making the steps in the algorithm easier to follow (the rule of thumb in chess is that you always assume that your opponent makes the best moves).

b) In the equation, if an error is made by moving both the variables and the numbers to the same side of the equation, a solution can still be found by moving either the variables or numbers back to the other side of the equation. It may also occur that two careless errors will occur in the same problem that may offset each other.

IV. Although not addressed in this paper, one may find shortcuts with a lot of practice.

a) The King may be used as a protecting piece for either the Queen or Rook. It would depend how close the white King is to the black King. The Rook and Queen can also protect each other so that a solution(checkmate) can be found much faster.

b) After completing steps in the algorithm, the equation would look like the linear equation $ax + b = c$. Solving this equation for x gives $–b/a$. This can be seen without any steps if the equation is written in this form. When students complete more problems, they start to look very similar to each other.

V. Small adjustments can be made to the algorithms listed above.

a) If the King is on 5^{th} through 8^{th} ranks, the algorithm can be adjusted accordingly. Also if the King is on the 1^{st} through 4^{th} file or if the King is on the 5^{th} through 8^{th} file, we can make small adjustments to the algorithm. The reason that the adjustments would be minimal is because of the symmetry of the chessboard.

b) In solving these equations, the steps change slightly depending on the textbook. Some textbooks might combine some of these steps together.